Cryptographic Complexity of Multi-party Computation Problems: Classifications and Separations

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Multi-Party Computation

Secure Multi-party Computation (MPC)

Parties engage in a protocol to securely accomplish some task, in the presence of adversaries.

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- Communication
- Function evaluation
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Fundamental Question

Which MPC tasks have secure protocols? (answer depends on MPC model)

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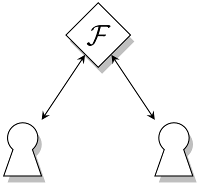

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- Parties: components for doing a task
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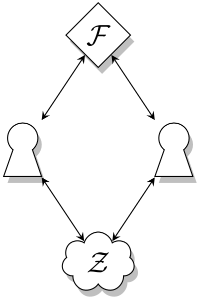

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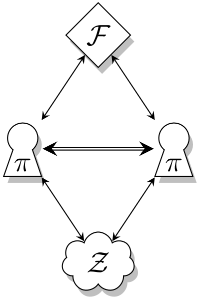

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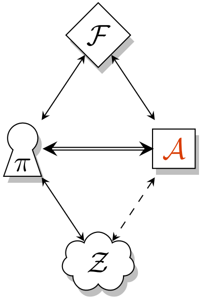
- Adversary: influences environment, corrupts parties



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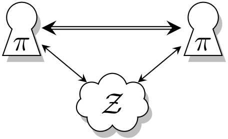

Real world interaction

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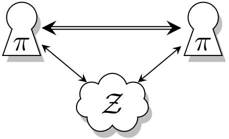

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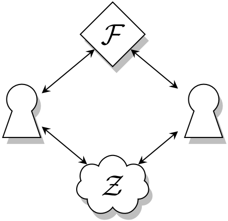

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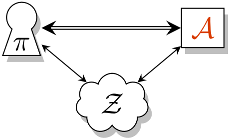

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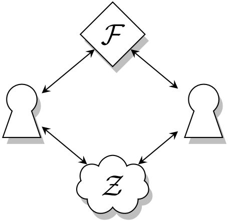

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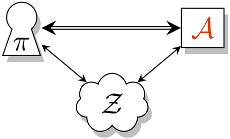

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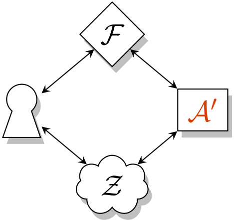

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- For every *real-world* adversary $\mathcal{A}$
- There exists an *ideal-world* adversary $\mathcal{A}'$
- Two worlds indistinguishable to *all* environments $\mathcal{Z}$

(Un)Feasibility Results in UC Framework

Main Question

Which functionalities are realizable in UC framework?

Ad hoc techniques (e.g., [C00,CF01]):

- Positive results (protocol constructions): e.g., can securely realize private channels using public channels
- Negative results (separations, impossibilities): bit commitment, ZK proofs, oblivious transfer, coin-tossing, etc...

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Towards more general-purpose techniques:

- Broad impossibility results for 2-party secure function evaluation [CKL03]
- Argument goes through even with certain "trusted set-up" functionalities [KL07]

Our Results

General-purpose tools to classify functionalities.

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Deviation-revealing property:

- Relate separations from passive corruption model
- Can make distinctions among higher-complexity functionalities

Outline

Motivation: Man-in-the-Middle Attacks

Implicit in all previous impossibility results in UC framework:

- Undetectable man-in-the-middle attack on protocol

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Key Idea

Make man-in-the-middle attack explicit in the *ideal* world:

- Against 2 instances of the functionality, not protocol

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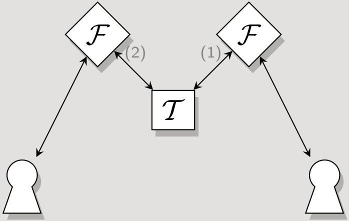

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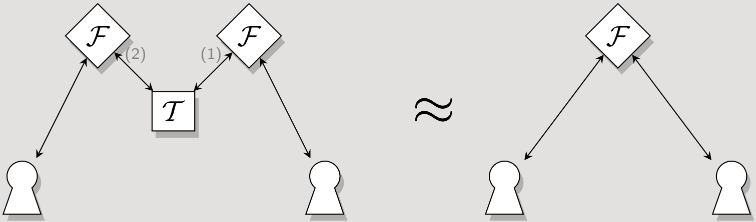

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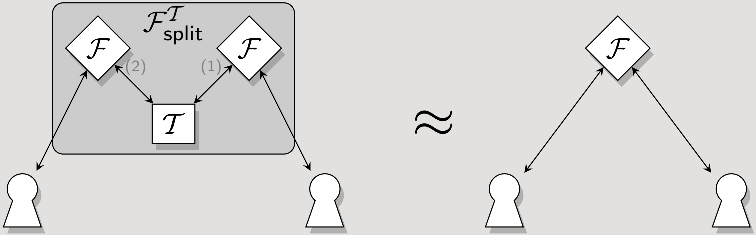

Formally, if $\mathcal{F}$ and $\mathcal{F}^{\mathcal{T}}_{\text{split}}$ indistinguishable for all environments.

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Can be very easy to show unsplittability.

Example: Coin-tossing functionality

Choose random $b \leftarrow \{0,1\}$, send b to both parties.

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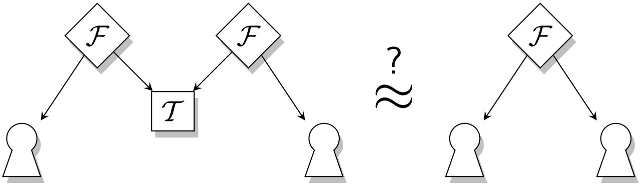

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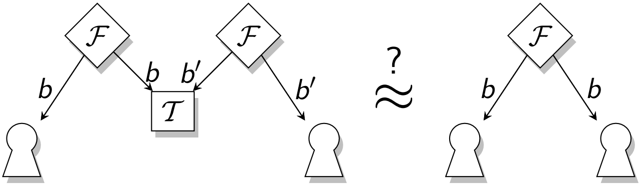

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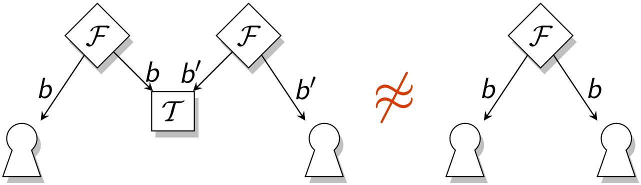

- Independent instances of $\mathcal{F}$ generate independent bits
- Environment can easily distinguish with probability $1/2$.

$\Longrightarrow$ Coin-tossing is not splittible.

Main Characterization

Also very easy to show unsplittable:

- Oblivious transfer
- Commitment
- ZK for NP $\setminus$ BPP

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Significance:

- First alternate characterization of realizability in UC model:
- Splittability defined in terms of black-box interactions with $\mathcal{F}$.
- $\mathcal{F}$ may be arbitrary (randomized, interactive, etc.)
- Can very easily re-derive essentially all UC impossibility results

Proof Overview

Main Theorem (one direction)

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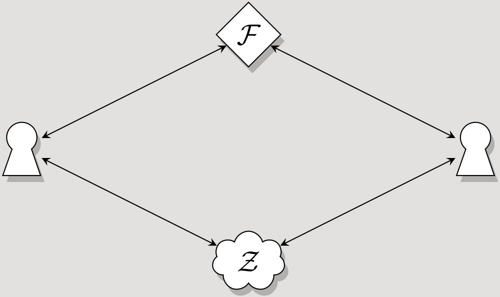

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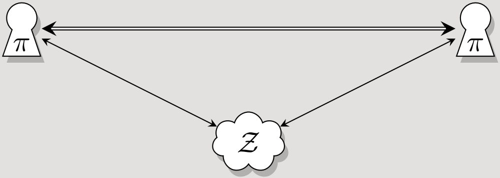

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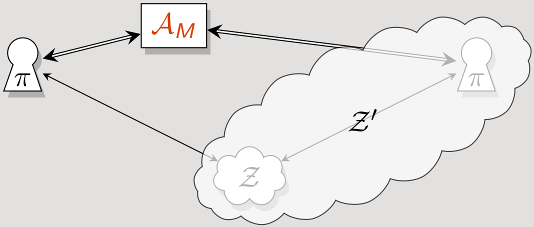

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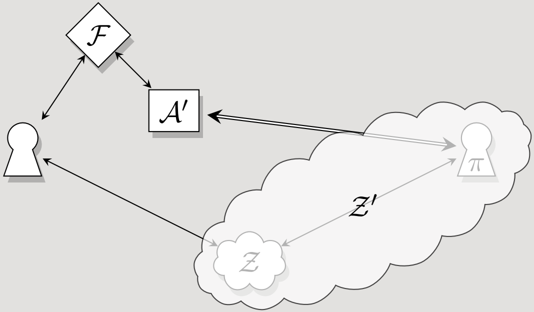

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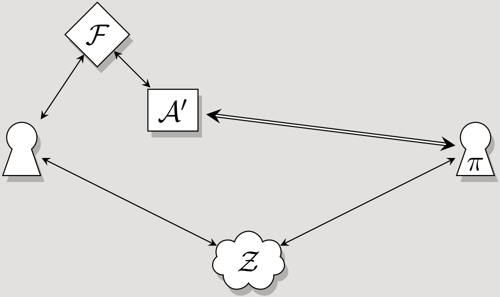

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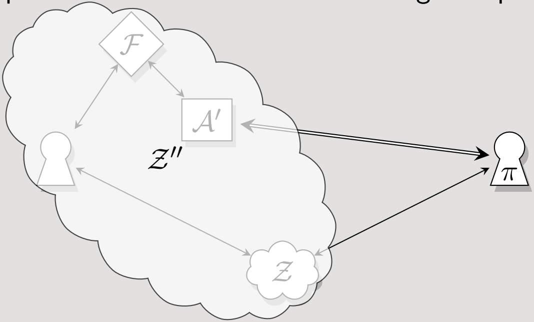

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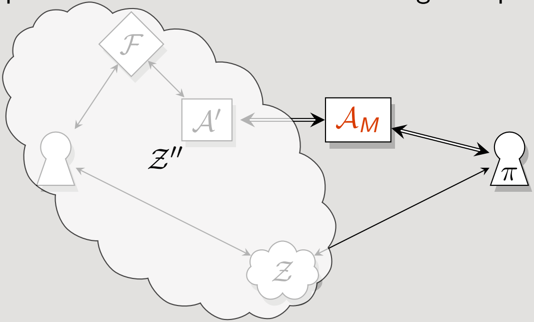

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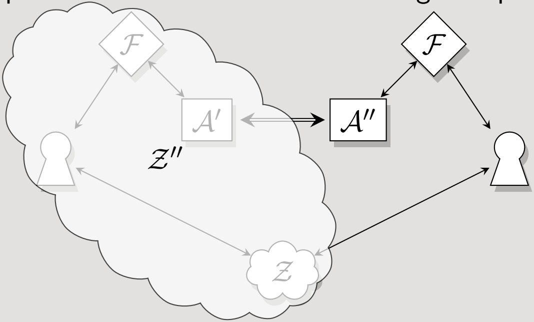

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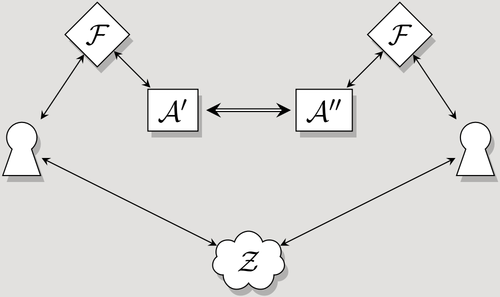

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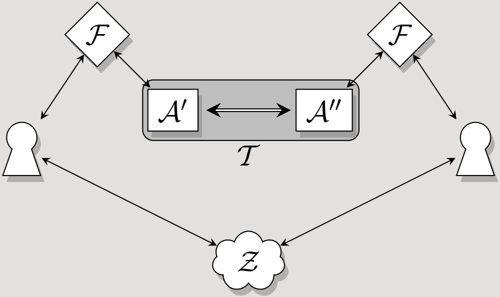

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Summary:

- Simple, unified paradigm for showing UC impossibility

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Have a characterization of realizability in UC framework, but..

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Goal:

- Build "complexity theory" for MPC tasks
- Understand structure of high-complexity tasks

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If there is a UC protocol for $\mathcal{F}$, is there necessarily a protocol for $\mathcal{F}$ secure against passive corruptions?

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Quiz

If there is a UC protocol for $\mathcal{F}$, is there necessarily a protocol for $\mathcal{F}$ secure against passive corruptions? No!

Counterexample $\mathcal{F}$: Receive bits x, y from Alice, Bob. Give $x \vee y$ to Bob.

- No protocol in passive model (unbounded parties)
- Secure UC protocol: Alice sends x to Bob
  - Bob could learn x in ideal world by sending $y = 0$.

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Notes:

- Property of $\mathcal{F}$, not the protocol!
- Definition applies to arbitrary $\mathcal{F}$

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When $\mathcal{F}$ is deviation-revealing, then separations in passive model imply separations in UC model.

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- Can now be translated to separations in UC model.

Applying this new technique in unbounded UC model:

- Can identify several intermediate levels of complexity
- Neither realizable nor complete

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Summary

New tools for analyzing complexity of UC functionalities:

- Apply to completely arbitrary functionalities

Apply new tools to obtain:

- Complete characterization of UC realizability
- Very easy paradigm for showing UC impossibility
- Combinatorial characterization for SFE
- Way to relate passive & active corruption settings

Future Work

Extend combinatorial characterizations to interactive functionalities.

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Classify all intermediate complexity levels (unbounded model)

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Characterize *completeness* of arbitrary functionalities:

- 0/1 Conjecture: Every functionality is either splittable or complete in PPT model.

fin.

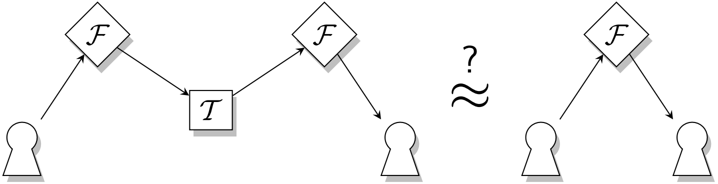
Thanks for your attention.

Special thanks to Qualcomm, PGP, and Marconi Society

Outline

Another Example

Bit commitment:



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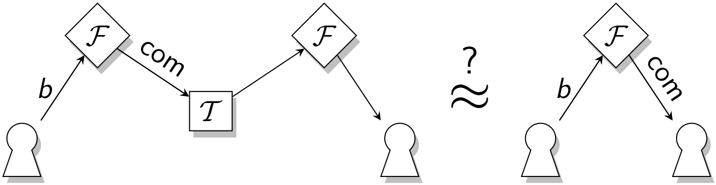

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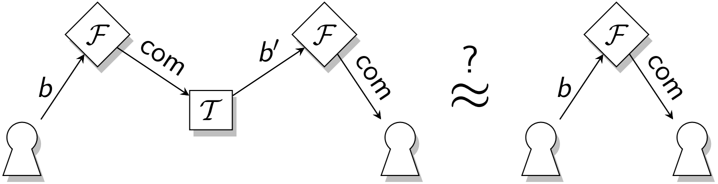

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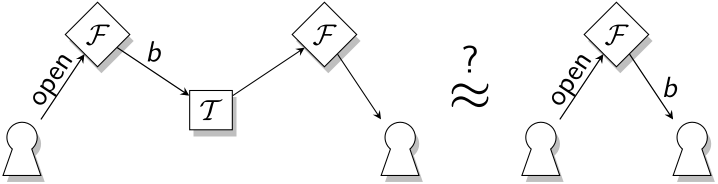

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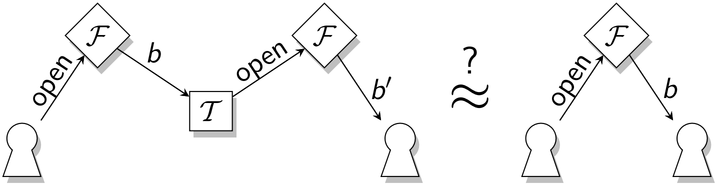

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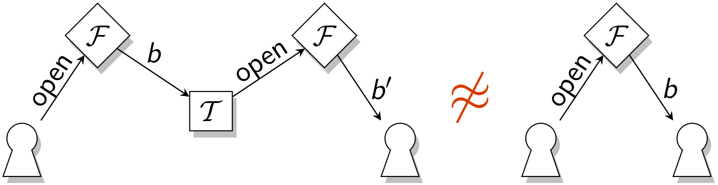

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$b \neq b'$ with probability $1/2$.